COST AND PROFIT ANALYSIS OF A NON-EMPTY QUEUE

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Abstract: *This paper deals with cost and profit analysis of a non-empty $M/M/1/N$ queuing system. A total cost function and total profit function are constructed and optimized with respect to both arrival and service parameters by using a fast converging Newton-Raphson's (abbreviated as N-R) method. The total optimal cost and profit of the model are computed by solving a system of two non-linear equations which are obtained by applying optimality conditions on the total cost function. Results are tabled and also presented graphically to better realize the performance of the system in different working conditions.*

Keywords: *Cost analysis, profit analysis, total optimal profit, optimal arrival rate, optimal service rate.*

1. INTRODUCTION

The non-empty $M/M/1/N$ queuing model is mostly applied in the field of inventory management, production management, computer and telecommunications, etc. The performance measures of the model can predict the efficiency, applicability, and quality of operating system of the non-empty $M/M/1/N$ queuing model. The applicability of the model depends on the total expected cost and profit of the system. The performance measures of the model can be easily evaluated by using standard results. But the main problem is to optimize these measures in such a way that the total expected cost and profit of the system are optimal with respect to the parameters, arrival and service rates. This problem can be solved only by using a powerful optimization technique with the help of computer and its scientific programming language. Till now, no serious attempt has been made in this direction.

Sharma and Tarabia [6] obtained the transient state probabilities for $M/M/1/N$ queuing system whence all particular cases concerning infinite waiting space and steady-state solutions can be derived straight away. Sharma and Gupta [7] discussed the transient behavior of the queue length of $M/M/1/N$ queue using Chebychev's polynomial. They expressed the transient state probabilities of the system free from Bessel's function which later led to the matrix method by Sharma [8].

Zhang et al. [5] developed a cost model for $M/M/1/N$ queuing system with balking, reneging, and server vacations and determined the optimal service rate. Taha [4] has discussed two queuing decision models namely, an aspiration level model and a cost model. Both models recognize the higher service levels reduce the waiting time in the system. He discussed the two conflicting costs viz. service cost and waiting cost and established a cost model. Mishra and Yadav [13] made an attempt on cost and profit analysis of single server Markovian queuing system with two priority classes. They constructed the functions of total expected cost, revenue, and profit of the system and optimized these functions with respect to service rates of lower and higher priority classes.

Abromovitz and Stegun [1] have introduced the fundamental concepts on cost analysis of various queuing models. Gross and Harris [2] have made an attempt on transient solution of $M/M/1/N$ queue, but the problem tedious when the restriction on waiting capacity is relaxed. Tarabia [10] has introduced an alternative simple approach, based on Laplace transform technique, to the study of transient behavior of non-empty $M/M/1/N$ queue. He has shown that the measures of effectiveness such as the first and second order moments of the queue length can be easily obtained in an elegant closed form. But he made no attempt to analyze the cost and profit of the model as very important aspects of the queuing system.

Takacs [9] obtained the transient solution for $M/M/1/N$ queuing system using eigen-vectors and eigen-values technique. Mishra and Pal [14] have introduced a computational approach to $M/M/1/N$ interdependent queuing system with controllable arrival rate. The computer coding in C programming language on the basis of algorithm have been developed to efficiently carry out the evaluation of performance measures of the model. They have presented the sensitivity analysis for the model in order to make it more efficient and applicable. Pern et al. [15] have considered the management policy of an $M/G/1$ queue with a single removable and non-reliable server. They applied an efficient Mat lab programmer to calculate optimal threshold of management policy and some system characteristics.

Ke [16] has studied the control policy of the N policy $M/G/1$ queue with server vacation, start up, and breakdowns where arrivals form a Poisson process and service times are generally distributed. He developed the total expected cost function per unit time to determine the optimal threshold of N at a minimum cost. Mishra and Yadav [12] analyzed the cost and profit for $M/E_k/1$ queuing model with removable service station under N-policy and steady state conditions. They introduced the notion of total revenue to find the total profit of the system with respect to total cost of the system.

Tarabia [11] obtained a new and simple series form for the transient state probabilities for non-empty $M/M/1/\infty$ queuing model. He has shown that the coefficients in this series satisfy iterative recurrence relations. Xu et al. [17] have discussed an $M/M/1$ queue with single working vacation and set-up times using quasi birth and death process and matrix-geometric solution method. They derived the distributions for the stationary queue length and waiting time of a customer in the system.

In this paper, we obtain various performance measures of the non-empty $M/M/1/N$ queue by programming in C++. We construct a total cost function and total profit function of the model and apply two-variable version of N-R method to obtain the optimal values of arrival rate λ and optimal service rate μ which optimize the total cost and profit functions. With optimal arrival and service rates, the performance measures like optimal expected number of customers in the system and optimal waiting time in the system are obtained. Finally, the numerical values are tabled and also presented in graphs to better understand the performance, applicability, and cost and profit level of the model.

2. COST ANALYSIS OF THE MODEL

The total cost function for this model is given by,

$$T_C = C_1\mu + C_2 L_s \tag{1}$$

where, T_C is the total cost of the system, C_1 is the service cost per customer per unit time, C_2 is waiting cost per customer per unit time, ρ is the traffic intensity of the system, and L_s is the expected number of customers in the system which is as given by Tarabia (2001),

$$L_s = \begin{cases} \dfrac{\rho\left[1-(N+1)\rho^N + N\rho^{N+1}\right]}{(1-\rho)\left(1-\rho^{N+1}\right)} & \text{if } \lambda \neq \mu \\ \dfrac{N}{2} & \text{if } \lambda = \mu \end{cases} \tag{2}$$

Where, N is the capacity of the system.

Therefore, from (1) and (2) we have

$$T_C = C_1\mu + C_2 \frac{\left[\rho-(N+1)\rho^{N+1}+N\rho^{N+2}\right]}{\left(1-\rho-\rho^{N+1}+\rho^{N+2}\right)}$$

$$= C_1\mu + C_2 \frac{\left[\dfrac{\lambda}{\mu}-(N+1)\left(\dfrac{\lambda}{\mu}\right)^{N+1}+N\left(\dfrac{\lambda}{\mu}\right)^{N+2}\right]}{\left[1-\dfrac{\lambda}{\mu}-\left(\dfrac{\lambda}{\mu}\right)^{N+1}+\left(\dfrac{\lambda}{\mu}\right)^{N+2}\right]}$$

Let $x=\left[\dfrac{\lambda}{\mu}-(N+1)\left(\dfrac{\lambda}{\mu}\right)^{N+1}+N\left(\dfrac{\lambda}{\mu}\right)^{N+2}\right]$, and

$$y=\left[1-\frac{\lambda}{\mu}-\left(\frac{\lambda}{\mu}\right)^{N+1}+\left(\frac{\lambda}{\mu}\right)^{N+2}\right].$$

Therefore,

$$T_C = C_1\mu + C_2\, x/y \tag{3}$$

Differentiating (3) partially, with respect to λ and μ, we get

$$\frac{\partial T_C}{\partial\lambda} = 0 + C_2 \frac{y\dfrac{\partial x}{\partial\lambda}-x\dfrac{\partial y}{\partial\lambda}}{y^2} \tag{4}$$

$$\frac{\partial T_C}{\partial\mu} = C_1 + C_2 \frac{y\dfrac{\partial x}{\partial\mu}-x\dfrac{\partial y}{\partial\mu}}{y^2} \tag{5}$$

For critical point $\left(\bar{\lambda}, \bar{\mu}\right)$, we must have $\dfrac{\partial T_C}{\partial\lambda}=\dfrac{\partial T_C}{\partial\lambda}=0$.

Therefore,

$$y\frac{\partial x}{\partial\lambda}-x\frac{\partial y}{\partial\lambda}=0 \tag{6}$$

$$C_1 y^2 + C_2\left\{y\frac{\partial x}{\partial\mu}-x\frac{\partial y}{\partial\mu}\right\}=0 \tag{7}$$

Now, to find $\dfrac{\partial x}{\partial\lambda}, \dfrac{\partial x}{\partial\mu}, \dfrac{\partial y}{\partial\lambda}$, and $\dfrac{\partial y}{\partial\mu}$ we proceed as follows:

Since $x=\left[\dfrac{\lambda}{\mu}-(N+1)\left(\dfrac{\lambda}{\mu}\right)^{N+1}+N\left(\dfrac{\lambda}{\mu}\right)^{N+2}\right]$,

$y=\left[1-\dfrac{\lambda}{\mu}-\left(\dfrac{\lambda}{\mu}\right)^{N+1}+\left(\dfrac{\lambda}{\mu}\right)^{N+2}\right]$ therefore,

$$\frac{\partial x}{\partial\lambda}=\frac{1}{\mu}\left[1-(N+1)^2\left(\frac{\lambda}{\mu}\right)^{N}+N(N+2)\left(\frac{\lambda}{\mu}\right)^{N+1}\right] \tag{8}$$

$$\frac{\partial y}{\partial\lambda}=\frac{1}{\mu}\left[-1-(N+1)\left(\frac{\lambda}{\mu}\right)^{N}+(N+2)\left(\frac{\lambda}{\mu}\right)^{N+1}\right] \tag{9}$$

$$\frac{\partial x}{\partial\mu}=\left(-\frac{\lambda}{\mu}\right)\frac{1}{\mu}\left[1-(N+1)^2\left(\frac{\lambda}{\mu}\right)^{N}+N(N+2)\left(\frac{\lambda}{\mu}\right)^{N+1}\right] \tag{10}$$

$$\frac{\partial y}{\partial\mu}=\left(-\frac{\lambda}{\mu}\right)\frac{1}{\mu}\left[-1-(N+1)\left(\frac{\lambda}{\mu}\right)^{N}+(N+2)\left(\frac{\lambda}{\mu}\right)^{N+1}\right] \tag{11}$$

From (8) and (10), we see that

$$\frac{\partial x}{\partial\mu}=\left(-\frac{\lambda}{\mu}\right)\frac{\partial x}{\partial\lambda} \tag{12}$$

From (9) and (11), we observe that

$$\frac{\partial y}{\partial\mu}=\left(-\frac{\lambda}{\mu}\right)\frac{\partial y}{\partial\lambda} \tag{13}$$

Let $W \equiv W(\lambda,\mu)=y\dfrac{\partial x}{\partial\lambda}-x\dfrac{\partial y}{\partial\lambda}$ and

$U \equiv U(\lambda,\mu)=C_1 y^2 + C_2\left\{y\dfrac{\partial x}{\partial\mu}-x\dfrac{\partial y}{\partial\mu}\right\}$.

Therefore from (6) and (7), we have

$$W(\lambda,\mu)=0 \text{ and } U(\lambda,\mu)=0 \tag{14}$$

The set of equations (14) represents a system of two non-linear equations in two variables λ and μ. We solve this system by applying a two variable version of N-R method, as discussed by Chapra and Canale [3], and the solution of this system will give critical point $\left(\bar{\lambda},\bar{\mu}\right)$. According to this method,

$$\Delta(\text{Jacobian})=\begin{vmatrix}\dfrac{\partial W_i}{\partial\lambda} & \dfrac{\partial W_i}{\partial\mu}\\ \dfrac{\partial U_i}{\partial\lambda} & \dfrac{\partial U_i}{\partial\mu}\end{vmatrix},\ \Delta_1=\begin{vmatrix}W_i & \dfrac{\partial W_i}{\partial\mu}\\ U_i & \dfrac{\partial U_i}{\partial\mu}\end{vmatrix},\ \Delta_2=\begin{vmatrix}\dfrac{\partial W_i}{\partial\lambda} & W_i\\ \dfrac{\partial U_i}{\partial\lambda} & U_i\end{vmatrix}$$

, $\lambda_{i+1}=\lambda_i-\dfrac{\Delta_1}{\Delta}$ and $\mu_{i+1}=\mu_i-\dfrac{\Delta_2}{\Delta}$, where (λ_i,μ_i) is the initial guess for (14), $W_i=W(\lambda_i,\mu_i)$

$U_i = U(\lambda_i, \mu_i)$, $\quad \dfrac{\partial W_i}{\partial \lambda} = \left(\dfrac{\partial W}{\partial \lambda}\right)_{(\lambda_i,\mu_i)}$, $\dfrac{\partial W_i}{\partial \mu} = \left(\dfrac{\partial W}{\partial \mu}\right)_{(\lambda_i,\mu_i)}$, $\dfrac{\partial U_i}{\partial \lambda} = \left(\dfrac{\partial U}{\partial \lambda}\right)_{(\lambda_i,\mu_i)}$, and $\dfrac{\partial U_i}{\partial \mu} = \left(\dfrac{\partial U}{\partial \mu}\right)_{(\lambda_i,\mu_i)}$. Therefore, we need to find $\dfrac{\partial W}{\partial \lambda}$, $\dfrac{\partial W}{\partial \mu}$, $\dfrac{\partial U}{\partial \lambda}$, and $\dfrac{\partial U}{\partial \mu}$.

Now,

$$\frac{\partial W}{\partial \lambda} = y\frac{\partial^2 x}{\partial \lambda^2} - x\frac{\partial^2 y}{\partial \lambda^2} \tag{15}$$

$$\frac{\partial W}{\partial \mu} = \left(\frac{1}{\lambda}\right)\left\{x\frac{\partial y}{\partial \mu} - y\frac{\partial x}{\partial \mu}\right\} + \left(\frac{\mu}{\lambda}\right)\left\{x\frac{\partial^2 y}{\partial \mu^2} - y\frac{\partial^2 x}{\partial \mu^2}\right\} \tag{16}$$

$$\frac{\partial U}{\partial \mu} = 2C_1\, y\frac{\partial y}{\partial \mu} + C_2\left\{y\frac{\partial^2 x}{\partial \mu^2} - x\frac{\partial^2 y}{\partial \mu^2}\right\} \tag{17}$$

$$\frac{\partial U}{\partial \lambda} = 2C_1\, y\frac{\partial y}{\partial \lambda} - \left(\frac{C_2}{\mu}\right)\left\{y\frac{\partial x}{\partial \lambda} - x\frac{\partial y}{\partial \lambda}\right\} - C_2\left(\frac{\lambda}{\mu}\right)\left\{y\frac{\partial^2 x}{\partial \lambda^2} - x\frac{\partial^2 y}{\partial \lambda^2}\right\} \tag{18}$$

From (15), (16), (17), and (18), we require $\dfrac{\partial^2 x}{\partial \lambda^2}$, $\dfrac{\partial^2 y}{\partial \lambda^2}$, $\dfrac{\partial^2 x}{\partial \mu^2}$, and $\dfrac{\partial^2 y}{\partial \mu^2}$

Therefore, differentiating (8), (9), (10), and (11) as follows:

$$\frac{\partial^2 x}{\partial \lambda^2} = \left[-N(N+1)^2\frac{\lambda^{N-1}}{\mu^{N+1}} + N(N+1)(N+2)\frac{\lambda^N}{\mu^{N+2}}\right]$$

$$\frac{\partial^2 y}{\partial \lambda^2} = \left[-N(N+1)\frac{\lambda^{N-1}}{\mu^{N+1}} + (N+2)(N+1)\frac{\lambda^N}{\mu^{N+2}}\right]$$

$$\frac{\partial^2 x}{\partial \mu^2} = \left[2\frac{\lambda}{\mu^3} - (N+2)(N+1)^2\frac{\lambda^{N+1}}{\mu^{N+3}} + N(N+2)(N+3)\frac{\lambda^{N+2}}{\mu^{N+4}}\right]$$

$$\frac{\partial^2 y}{\partial \mu^2} = \left[-2\frac{\lambda}{\mu^3} - (N+1)(N+2)\frac{\lambda^{N+1}}{\mu^{N+3}} + (N+2)(N+3)\frac{\lambda^{N+2}}{\mu^{N+4}}\right]$$

The total cost function T_C will be optimum at the critical point $(\bar{\lambda}, \bar{\mu})$ if (I) $\begin{vmatrix} \dfrac{\partial^2 T_C}{\partial \lambda^2} & \dfrac{\partial^2 T_C}{\partial \lambda \partial \mu} \\ \dfrac{\partial^2 T_C}{\partial \mu \partial \lambda} & \dfrac{\partial^2 T_C}{\partial \mu^2} \end{vmatrix} > 0$, (II) $\dfrac{\partial^2 T_C}{\partial \lambda^2} > 0$.

After calculating the optimal arrival and service rates $\bar{\lambda}$ and $\bar{\mu}$ respectively, we find the performance measures of the system which are optimal expected number of customers in the system $\bar{L}_s$ and optimal waiting time in the system $\bar{W}_s$ by applying Little's law, which states that $\bar{L}_s = \bar{\lambda}\bar{W}_s$ and computer programming in C++.

Now, we shall find second order partial derivatives of T_C appeared in (I) and (II). Differentiating equations (4) and (5) partially with respect to λ and μ, we get

$$\frac{\partial^2 T_C}{\partial \lambda^2} = C_2\left[\frac{y\dfrac{\partial^2 x}{\partial \lambda^2} - x\dfrac{\partial^2 y}{\partial \lambda^2}}{y^2} - \frac{2\dfrac{\partial y}{\partial \lambda}\left\{y\dfrac{\partial x}{\partial \lambda} - x\dfrac{\partial y}{\partial \lambda}\right\}}{y^3}\right]$$

$$\frac{\partial^2 T_C}{\partial \mu^2} = C_2\left[\frac{y\dfrac{\partial^2 x}{\partial \mu^2} - x\dfrac{\partial^2 y}{\partial \mu^2}}{y^2} - \frac{2\dfrac{\partial y}{\partial \mu}\left\{y\dfrac{\partial x}{\partial \mu} - x\dfrac{\partial y}{\partial \mu}\right\}}{y^3}\right]$$

Using (12) and (13) in (4), we get

$$\frac{\partial T_C}{\partial \lambda} = C_2\left(-\frac{\mu}{\lambda}\right)\frac{y\dfrac{\partial x}{\partial \mu} - x\dfrac{\partial y}{\partial \mu}}{y^2}$$

Therefore from (5), we have $\dfrac{\partial T_C}{\partial \lambda} = C_2\left(-\dfrac{\mu}{\lambda}\right)\left(\dfrac{\partial T_C}{\partial \mu} - C_1\right)$

$$\frac{\partial^2 T_C}{\partial \mu \partial \lambda} = \left(-\frac{1}{\lambda}\right)\left(\frac{\partial T_C}{\partial \mu} - C_1\right) + \left(-\frac{\mu}{\lambda}\right)\frac{\partial^2 T_C}{\partial \mu^2}$$

Using (12) and (13) in (5), we get

$$\frac{\partial T_C}{\partial \mu} = C_1 + C_2\left(-\frac{\lambda}{\mu}\right)\frac{y\dfrac{\partial x}{\partial \lambda} - x\dfrac{\partial y}{\partial \lambda}}{y^2}$$

Therefore from (4), we have $\dfrac{\partial T_C}{\partial \mu} = C_1 + \left(-\dfrac{\lambda}{\mu}\right)\dfrac{\partial T_C}{\partial \lambda}$

$$\frac{\partial^2 T_C}{\partial \lambda \partial \mu} = \left(-\frac{1}{\mu}\right)\frac{\partial T_C}{\partial \lambda} + \left(-\frac{\lambda}{\mu}\right)\frac{\partial^2 T_C}{\partial \lambda^2}$$

3. PROFIT ANALYSIS OF THE MODEL

We now find the total expected profit (T_P) of the system on the basis of the total revenue earned by the system in rendering its service to the customers.

Suppose that R is the earned revenue for providing the service to each customer then total expected revenue (T_R) of the system is given by, $T_R = R L_s$. From (1), total cost of the system is $T_C = C_1\mu + C_2 L_s$. Therefore total expected profit of the system will be

$$T_P = T_R - T_C = R L_s - (C_1\mu + C_2 L_s) = (R - C_2)L_s - C_1\mu \tag{19}$$

The optimal arrival and service rates $\bar{\lambda}$ and $\bar{\mu}$ respectively optimize the total expected profit of the system given by (19). We evaluate the total optimal profit of the system $\bar{T}_P$ and analyze the effect of variations in parameters on it by developing a computing algorithm in C++.

4. SENSITIVITY ANALYSIS OF THE MODEL

A computing algorithm in C++ is developed to compute the optimal arrival and service rates $\bar{\lambda}$ and $\bar{\mu}$ respectively which optimize the total cost T_C of the system and total expected profit T_P of the system which are given by (1) and (19) respectively. The performance measures of the system $\bar{L}_s$ and $\bar{W}_s$ are also computed with the help of computing algorithm. The changes in these performance measures of the system with respect to variations in the parameters waiting cost, service cost, and capacity of the system are putted in various tables.

The outcomes are also presented in graphs to exhibit the correlation between these parameters and performance measures. Observations are drawn on the basis of existing tables and graphs to better realize the efficiency and performance of the system in different circumstances.

Table no 1: Service Cost vs. various Performance Measures

$N = 40,\ C_2 = 3.80$

(C_1)	$(\bar{\lambda})$	$(\bar{\mu})$	$\bar{L}_s$	$\bar{W}_s$	$(\bar{T_C})$
3.00	3.86	3.91	18.2	4.72	80.91
4.00	3.84	3.89	18.2	4.74	84.71
5.00	3.82	3.87	18.2	4.76	88.46
6.00	3.81	3.86	18.2	4.78	92.26
7.00	3.80	3.85	18.2	4.79	96.03
8.00	3.80	3.84	18.5	4.87	101.17
9.00	3.80	3.84	18.5	4.87	105.01
10.00	3.79	3.83	18.5	4.88	108.73
11.00	3.78	3.82	18.5	4.89	112.44
12.00	3.78	3.82	18.5	4.89	116.26
13.00	3.77	3.82	18.2	4.83	118.68
14.00	3.77	3.81	18.2	4.91	123.74

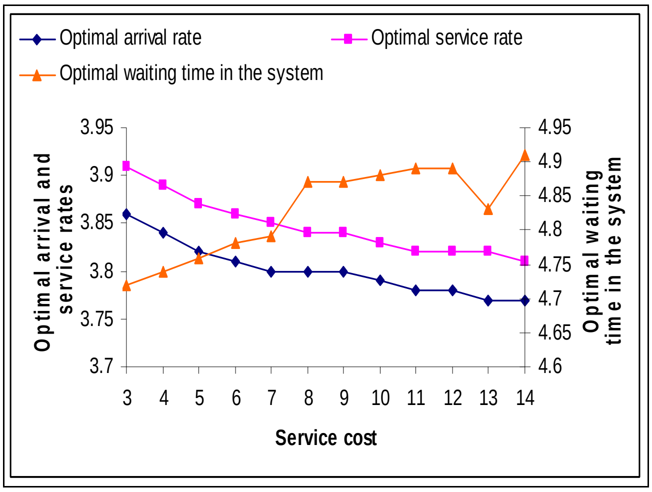

Fig. 1.1: Service Cost vs. Optimal Waiting Time in the System

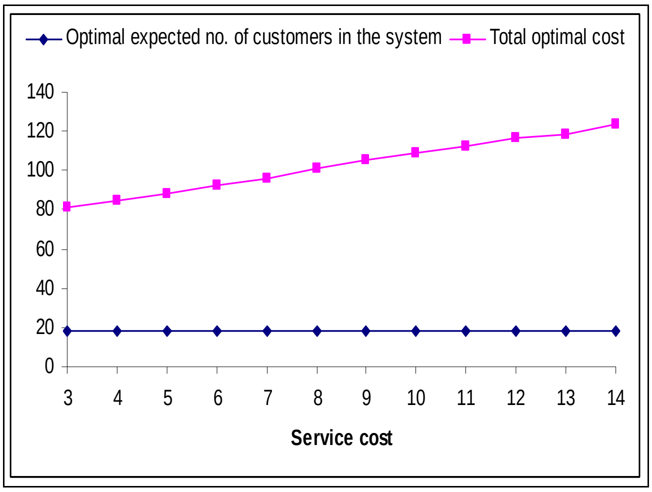

Fig. 1.2: Service Cost vs. Total Optimal Cost

Table no 2: Waiting Cost vs. various Performance Measures

$N = 40,\ C_1 = 3.00$

(C_2)	$(\bar{\lambda})$	$(\bar{\mu})$	$\bar{L}_s$	$\bar{W}_s$	$(\bar{T_C})$
3.80	3.86	3.91	18.2	4.72	80.91
4.80	3.88	3.93	18.2	4.69	99.22
5.80	3.90	3.95	18.2	4.67	117.55
6.80	3.91	3.96	18.2	4.65	135.84
7.80	3.92	3.97	18.2	4.64	154.13
8.80	3.93	3.98	18.2	4.63	172.43
9.80	3.94	3.99	18.2	4.62	190.75
10.80	3.94	4.00	17.9	4.54	205.29
11.80	3.95	4.00	18.2	4.61	227.31
12.80	3.95	4.01	17.9	4.53	241.18
13.80	3.96	4.02	17.9	4.52	259.20
14.80	3.96	4.02	17.9	4.52	277.10

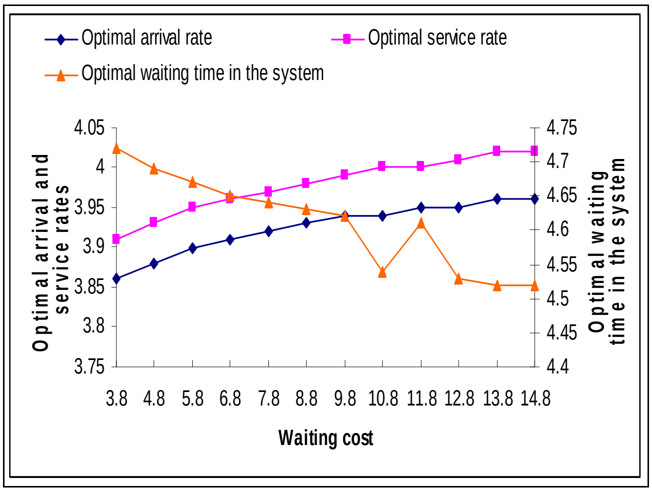

Fig. 2.1: Waiting Cost vs. Optimal Waiting Time in the System

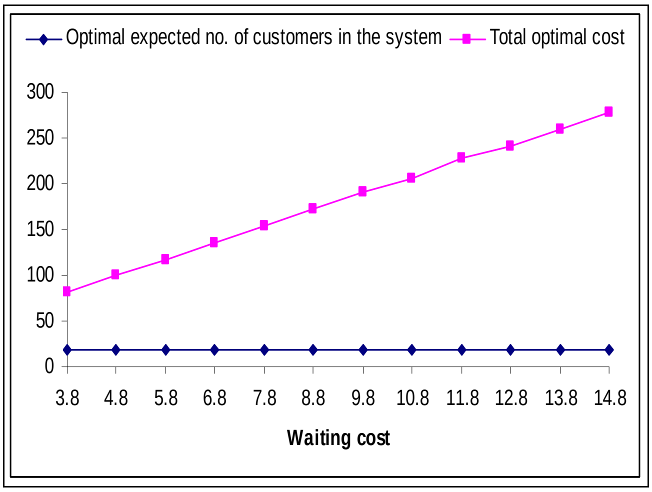

Fig. 2.2: Waiting Cost vs. Total Optimal Cost

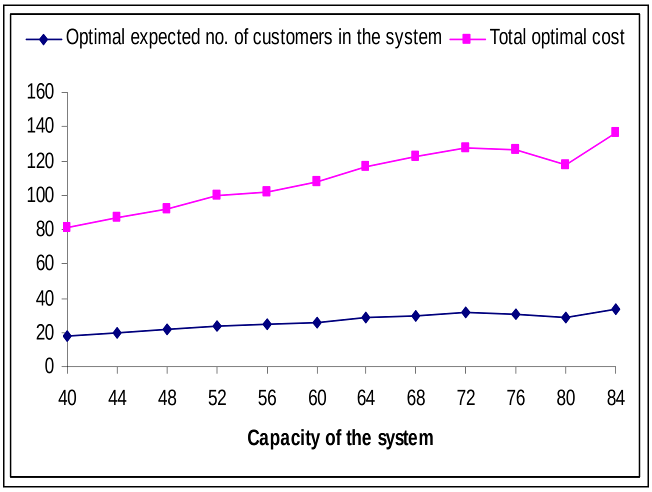

Fig. 3.2: Capacity of the System vs. Total Optimal Cost

Table no 3: Capacity of the System vs. various Performance Measures

$C_1 = 3.00,\ C_2 = 3.80$

N	$(\bar{\lambda})$	$(\bar{\mu})$	$\bar{L}_s$	$\bar{W}_s$	$(\bar{T}_C)$
40	3.86	3.91	18.2	4.72	80.91
44	3.42	3.46	20.0	4.69	86.56
48	3.21	3.25	21.5	4.67	91.60
52	3.09	3.12	23.7	4.65	99.61
56	3.00	3.04	24.4	4.64	102.02
60	2.94	2.98	25.9	4.63	107.20
64	2.90	2.93	28.4	4.62	116.73
68	2.86	2.89	29.9	4.54	122.28
72	2.83	2.86	31.4	4.61	127.76
76	2.80	2.84	31.1	4.53	126.81
80	2.76	2.82	28.8	4.52	117.90
84	2.76	2.80	33.5	4.52	135.90

Table no 4: Service Cost vs. Total Optimal Profit

$N = 45,\ C_2 = 3.25$

(C_1)	$(\bar{\lambda})$	$(\bar{\mu})$	$(\bar{T}_P)$
6.50	4.32	3.88	768.93
7.50	4.32	3.88	765.05
8.50	4.32	3.88	761.17
9.50	4.32	3.88	757.29
10.50	4.32	3.88	753.41
11.50	4.32	3.88	749.53
12.50	4.32	3.88	745.65
13.50	4.32	3.88	741.77
14.50	4.32	3.88	737.89
15.50	4.32	3.88	734.01
16.50	4.32	3.88	730.13
17.50	4.32	3.88	726.25
18.50	4.32	3.88	722.37
19.50	4.32	3.88	718.49

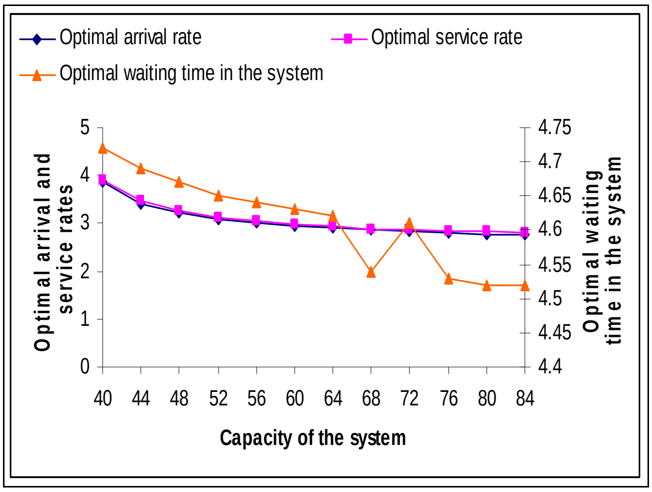

Fig. 3.1: Capacity of the System vs. Optimal Waiting Time in the System

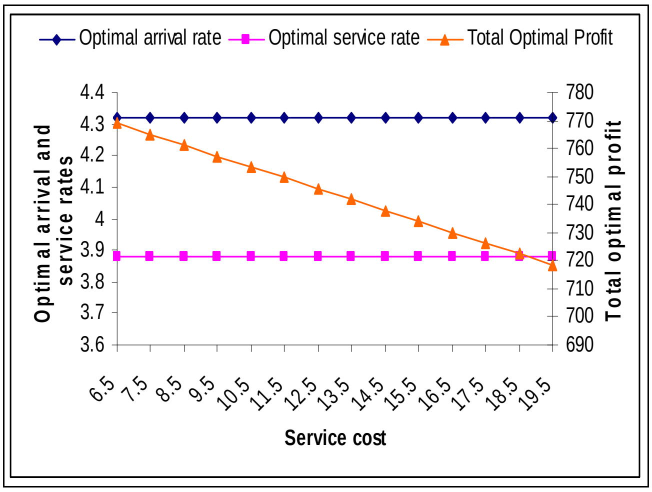

Fig. 4: Service Cost vs. Total Optimal Profit

Table no 5: Waiting Cost vs. Total Optimal Profit

$N = 45,\ C_1 = 6.50$

(C_2)	$(\bar{\lambda})$	$(\bar{\mu})$	$(\bar{T}_P)$
4.50	4.32	3.88	723.20
5.50	4.32	3.88	686.78
6.50	4.32	3.88	650.27
7.50	4.32	3.88	613.75
8.50	4.32	3.88	577.24
9.50	4.32	3.88	540.73
10.50	4.32	3.88	504.22
11.50	4.32	3.89	467.70
12.50	4.32	3.89	431.19
13.50	4.32	3.89	394.68
14.50	4.32	3.89	358.17
15.50	4.32	3.89	321.65
16.50	4.32	3.89	285.14
17.50	4.32	3.89	248.63

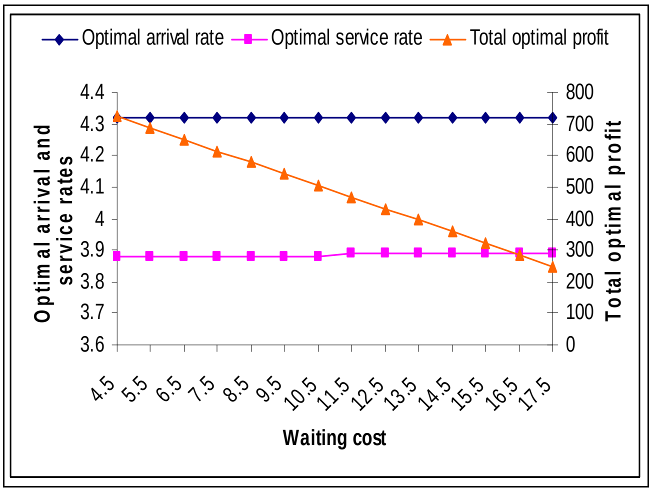

Fig.5: Waiting Cost vs. Total Optimal Profit

Table no 6: Capacity of the System vs. Total Optimal Profit

$C_1 = 6.50,\ C_2 = 3.50$

N	$(\bar{\lambda})$	$(\bar{\mu})$	$(\bar{T}_P)$
45	4.32	3.88	759.81
50	4.32	3.88	864.79
55	4.32	3.88	970.63
60	4.32	3.88	1077.06
65	4.32	3.88	1183.87
70	4.32	3.88	1290.93
75	4.32	3.88	1398.15
80	4.32	3.87	1510.18
85	4.32	3.87	1617.59
90	4.32	3.87	1725.03
95	4.32	3.87	1832.50
100	4.32	3.87	1939.98
105	4.32	3.87	2047.46
110	4.32	3.87	2154.96

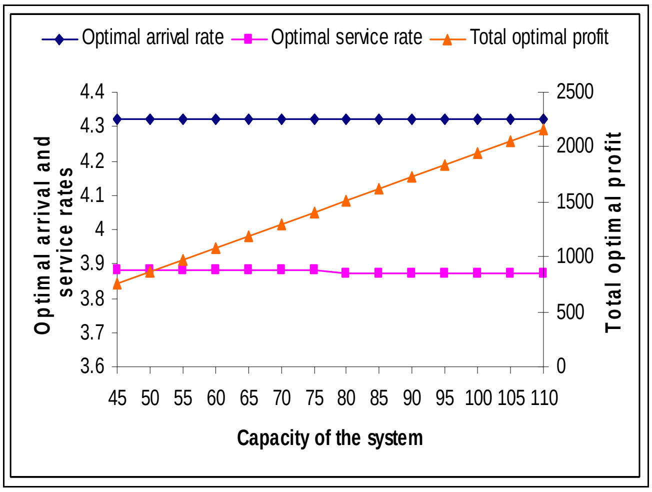

Fig. 6: Capacity of the System vs. Total Optimal Profit

Table no 7: Earned Revenue vs. Total Optimal Profit

R	N	(C_1)	(C_2)	$(\bar{T}_P)$
20.00	75	6.50	3.50	1070.74
22.00	75	6.50	3.50	1203.57
24.00	75	6.50	3.50	1336.41
26.00	75	6.50	3.50	1469.25
28.00	75	6.50	3.50	1602.08
30.00	75	6.50	3.50	1734.92
32.00	75	6.50	3.50	1867.75
34.00	75	6.50	3.50	2000.59
36.00	75	6.50	3.50	2133.42
38.00	75	6.50	3.50	2266.26
40.00	75	6.50	3.50	2399.09
42.00	75	6.50	3.50	2531.93
44.00	75	6.50	3.50	2664.77
46.00	75	6.50	3.50	2797.60

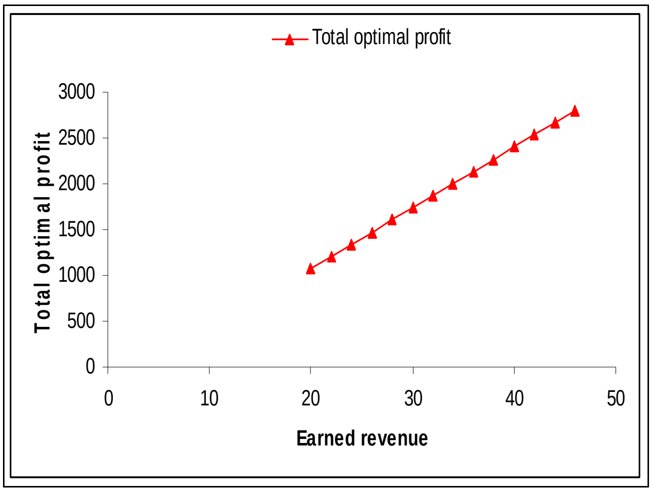

Fig. 7: Earned Revenue vs. Total Optimal Profit

Observations: In Figure 1.1, we observe that the optimal arrival and service rates decrease very slowly but optimal waiting time in the system shows increasing trend with fluctuations as service cost increases. Therefore service cost and optimal waiting time in the system are in positive correlation. In Figure 1.2 we see that total optimal cost of the system increases as service cost increases. In fact about 16.6% increase in service cost causes approx. 4.1% increase in the total optimal cost of the system. Thus service cost and total optimal cost of the system are in positive correlation. The optimal expected number of customers in the system remains constant as service cost increases.In Figure 2.1, we see that the optimal arrival and service rates increase gradually as waiting cost increases. The optimal waiting time in the system shows decreasing trend with fluctuations in the end as waiting cost increases. Thus waiting cost and optimal waiting time in the system are in negative correlation. In Figure 2.2, we observe that the total optimal cost of the system increases as waiting cost increases. An increase of 17.2% (approx.) in waiting cost results about 15.6% increase in the total optimal cost of the system. Thus waiting cost and total optimal cost of the system are in positive correlation. The optimal expected number of customers in the system does not vary as waiting cost increases.In Figure 3.1, we observe that the optimal arrival and service rates decreases very slowly as capacity of the system increases. The optimal waiting time in the system shows a decreasing trend with some fluctuations as capacity of the system increases. Thus capacity of the system and optimal waiting time in the system are in negative correlation. In Figure 3.2, we see that the total optimal cost of the system and optimal expected number of customers in the system increase gradually as capacity of the system increases. In fact approx. 7.7% increase in the capacity of the system causes approx. 3% increase in optimal expected number of customers in the system and 2.6% increase in total optimal cost of the system.In Figure 4, we observe that the total optimal profit of the system decreases as service cost increases. In fact, about 13.3% increase in service cost causes 0.5% decrease in total optimal profit of the system. Thus a very weak negative correlation between service cost and total optimal profit is seen. The optimal arrival and service rates do not vary as service cost increases.In Figure 5, we see that as waiting cost increases the total optimal profit of the system decreases considerably. In fact, about 5.3% decrease in total optimal profit is observed due to about 18.2% increase in waiting cost. There is a negative correlation between waiting cost and total optimal profit of the system. The optimal arrival and service rates remain constant when waiting cost increases. In Figure 6, we observe that the total optimal profit of the system increases as capacity of the system increases. An increase of 5 units in the capacity of the system results about 12.2% increase in total optimal profit of the system. The optimal arrival and service rates are constant when capacity of the system increases.In Figure 7, we see that as earned revenue increases the total optimal profit of the system increases rapidly. About 7.7% increase in the earned revenue gives approx. 9.1% increase in total optimal profit of the system. Thus earned revenue and total optimal profit of the system are in positive correlation.

5. CONCLUSION

Here, we have succeeded in presenting the cost and profit analysis of non-empty $M/M/1/N$ queuing system. This has led us to efficiently evaluate optimal arrival rate, optimal service rate, and optimal expected number of customers in the system, optimal waiting time in the system, total optimal cost of the system, and total optimal profit of the system as important performance measures of the system. The problem has good deal of potential to the applications in various fields including inventory management, computer and telecommunications, production management etc.

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MAPPING MACROECONOMIC TIME SERIES INTO WEIGHTED NETWORKS

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Abstract. *The correlations between GDP/capita growth rates of 27 European countries are scanned in various moving time window sizes. The square averaged correlation coefficients are taken as the link weights for a network having the countries as vertices. The network average degree and the weight set variance are found to be monotonic functions on the time window size. The statistics of the weight distributions as well as the adjacency matrix eigensystem are discussed. A new measure of the so called country overlapping is proposed and applied to the network. The ties and clusters are better emphasized through a threshold analysis. The derived clustering structure is found to confirm intuitive or empirical aspects, like the convergence clubs i.e. have a remarkable consistency with the results reported in the actual economic literature.*

Keywords: *fluctuations, correlations, network, clusters*
PACS numbers: 89.65.Gh, 89.75.Fb, 05.45.Tp

1. INTRODUCTION

Modelling the dependences between the macroeconomic (ME) variables has to take into account circumstances that differ substantially from those encountered in the natural sciences. First, experimentation is usually not feasible and is replaced by survey research, implying that the explanatory variables cannot be manipulated and fixed by the researcher. Second, the number of possible explanatory variables is often quite large, unlike the small number of carefully chosen treatment variables frequently found in the natural sciences. Third, the ME time series are short and noisy. Most data have a yearly frequency. When social time series have been produced for a very long period, there is usually strong evidence against stationarity.

Some macroeconomic (ME) indicators are monthly and/or quarterly registered, increasing in this way the number of available data points, but some additional noise is naturally enclosed in the time series so generated (seasonal fluctuations, external and internal short range shocks, etc). This seems to be a solid argument for the fact that the main data sources, at least the ones freely available on the web, tend only to keep the annual averages/rates of growth of the ME indicators.

Let us consider, for example, a time interval of one hundred years, which is mapped onto a graphical plot of 100 data points. From the statistical physics viewpoint, 100 is a quite *small* number of data points, surely too small for speaking about the so called "thermodynamic limit". On the other hand, from a socio-economic point of view, we can justifiably wonder if a growth, say, of 2% of any ME indicator has at the present time the same meaning as it had one century ago. One must take into account that during that time, the social, politic and economic environment was drastically changed. Moreover the methodology of data collecting and processing is today different from what it was two generations ago. Indeed, the economic world is created by people and is substantially changing from a generation to another one (sometimes also during one and the same generation). Thus, this way of statistical data aggregation turns to be controversial.

On the other hand, an increasing interest in network analysis has been registered during the last decade, particularly due to its potential unbounded area of application. Indeed, the inter-disciplinary (or rather trans-disciplinary) concept of "network" is frequently met in all scientific research areas, its covering field spanning from the computer science to the medicine and social psychology. Moreover it proves to be a reliable bridge between the natural and social sciences, so the recent interest in this field is fully justified.

Using the strong methodological arsenal of the mathematical graph theory, the physicists mainly focused on the *dynamical* evolution of networks, *i.e.* on the statistical physics of growing networks. The remarkable extension from the concept of classical random graph [1] to the one of non-equilibrium growing network [2] allows for accounting the structural properties of random complex networks in communications, biology, social sciences and economics [3, 4]. Indeed, the field of the possible applications seems to be unbounded, it spanning from the "classical" WWW and Internet structures [5, 6] to some more sophisticated social networks of scientific collaborations [7-9], paper citations [10] or collective listening habits and music genres [11].

In most approaches, the Euler graph theory legacy was preserved, especially as regards to the "Boolean" character of links: two vertices can only be either tied or not tied, thus the elements of the so-called adjacency matrix only consist of zeros and ones. However, many biological and social networks, and particularly almost all economic networks, must be characterised by *different* strengths of the links between vertices. This aspect led to the concept of "weighted network" as a natural generalisation of the graph-like approaches. Of course, various ways of attaching some weights to the edges of a fully connected network [12-14]. Some ways to relate the weights to the correlations between various properties of nodes have been proposed in the recent literature [15-18].

The correlation coefficients C_{ij} between two ME time series $\{x_i\}$ and $\{y_j\}$, i, j = 1, …, N, is calculated in the present work according to the (Pearson's) classical formula:

$$C_{ij}(t,T) = \frac{< x_i y_j > - < x_i >< y_j >}{\sqrt{< x_i^2 - < x_i >^2 >< y_j^2 - < y_j >^2 >}} \qquad (1)$$